Fast and Accurate Solution of Time Domain Electric Field Integral Equation for Dielectric Half-Space

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1. Introduction

Fast and accurate computational schemes for analyzing broadband electromagnetic phenomena in layered media are essential in the design of printed circuit boards, microstrip antennas, and monolithic microwave integrated circuits. Often, method of moment based frequency domain integral equation solvers are used for this purpose [1,2]. This is evidenced by the multitude of mature frequency domain method of moments based CAD packages.

Layered-medium scattering and radiation problems can also be analyzed using marching-on-in-time (MOT) based time domain integral equation (TDIE) solvers. Especially when broadband data is required or when the device under study is nonlinear, this avenue is appealing. Unfortunately, the use of layered-media Green’s functions in MOT solvers drastically increases both the matrix-fill and marching times. Indeed, because of the temporal tail of the layered-medium Green’s function, the classical MOT analysis of scattering and radiation phenomena in layered media requires $O(N_t^2 N_s^2)$ operations and $O(N_t N_s^2)$ storage as opposed to $O(N_t N_s^2)$ operations and $O(N_s^2)$ memory for free space problems [3]. Here, $N_t$ is the number of time steps and $N_s$ is the number of spatial samples in the analysis.

In this work, an FFT-accelerated MOT solver [3-5] is used to analyze broadband scattering phenomena in a half-space environment using only $O(N_t N_s \log(N_t N_s) \log N_t)$ CPU and $O(N_t N_s)$ memory resources. The multilevel space-time FFT scheme accelerates both the matrix-fill and marching times by exploiting the translational invariance of the underlying integral kernel through the use of uniform spatial meshes. All time domain Green’s functions are computed by inverse Fourier transforming their frequency domain counterparts, which, in turn, are computed by adaptive evaluation of Sommerfeld integrals. In what follows, the proposed method is applied to the analysis of transient scattering from PEC bodies residing above a dielectric half-space. The method is flexible enough to be used in the analysis of layered media with more complicated Green’s functions and can be extended to nonuniformly meshed scatterers using the time domain adaptive integral method [6].

2. Formulation

Let $S$ denote the surface of a perfect electric conductor that resides in a first dielectric medium with permittivity $\varepsilon=\varepsilon_r \varepsilon_0$ above a half-space comprised of a second medium with relative permittivity $\varepsilon_r$ (Fig. 1). Both media have constant permeability $\mu$. An
incident electric field $\mathbf{E}^{inc}(\mathbf{r},t)$ induces a surface current density $\mathbf{J}(\mathbf{r},t)$ on $S$. This current satisfies the following electric field integral equation:

$$
\partial_t \mathbf{E}^{inc}(\mathbf{r},t)_{\text{tan}} = \left[ \mu \left[ \int_S \mathbf{E}^{inc}(\mathbf{r},t)/(4\pi R)ds' - \nabla' \cdot \mathbf{J}(\mathbf{r}',\tau)/(4\pi \varepsilon)ds' + \sum_{l-t}^{t_{\text{min}}} \int_S \mathbf{G}^{\text{ref}}(\mathbf{r},\mathbf{r}', t-t') \cdot \mathbf{J}(\mathbf{r}', t')ds'dt' \right] \right].
$$

(1)

Here, $\mathbf{E}^{inc}$ is the incident field, $\mathbf{J}$ is the induced current density, $\mathbf{G}^{\text{ref}}$ is the reflected part of the time domain dyadic Green’s function, and $t_{\text{min}}$ is the minimum propagation time for the reflected field between any two points on $S$. Equation (1) is solved by expanding $\mathbf{J}(\mathbf{r},t)$ as

$$
\mathbf{J}(\mathbf{r},t) = \sum_{k' \neq l} \sum_{l=1}^{N_l} I_{k',l} S_{k'}(\mathbf{r}) T(t-t'\Delta t),
$$

(2)

where $I_{k',l}$ are unknown weighting coefficients related to the space-time basis functions $S_{k'}(\mathbf{r}) T(t-t'\Delta t)$, and $\Delta t$ is the time step. To find $I_{k',l}$, Eq. (2) is inserted into Eq. (1) and the resulting equation is converted into the following matrix system by Galerkin testing in space and point matching in time:

$$
\mathbf{Z}_l \mathbf{I}_l = \mathbf{V}_l^{\text{inc}} - \sum_{l'=1}^{l-1} \mathbf{Z}_{l-l'} \mathbf{I}_{l'}, \quad \text{for } l=1,\ldots,N_l.
$$

(3)

Here, $\mathbf{V}_l^{\text{inc}}$ and $\mathbf{I}_l$ are the tested incident field and the unknown coefficient vectors at time step $l$, respectively. The impedance matrices can be decomposed as $\mathbf{Z}_{l-l'} = \mathbf{Z}_{l-l'}^{\text{direct}} + \mathbf{Z}_{l-l'}^{\text{ref}}$. The matrices $\mathbf{Z}_{l-l'}^{\text{direct}}$ are the same as those in [4], whereas the entries of $\mathbf{Z}_{l-l'}^{\text{ref}}$ are given by

$$
\mathbf{Z}_{l-l'}^{\text{ref}}(k,k') = \int_S \int_S \mathbf{S}_k(\mathbf{r}) \mathbf{C}(\mathbf{r},\mathbf{r}',(l-l')\Delta t) \mathbf{S}_{k'}(\mathbf{r}') ds'ds,
$$

(4)

where the dyad $\mathbf{C}$ is the convolution of $\mathbf{G}^{\text{ref}}$ with the time derivative of $T(t)$:

$$
\mathbf{C}(\mathbf{r},\mathbf{r}',(l-l')\Delta t) = \int_0^{(l-l')\Delta t_{\text{min}}} \mathbf{G}^{\text{ref}}(\mathbf{r},\mathbf{r}',(l-l')\Delta t-t') \partial_{t'} T((l-l')\Delta t) dt'.
$$

(5)

Equations (3)-(5) show that because of the temporal convolutions, filling the MOT matrices becomes significantly more costly when the Green’s function has a tail. While the convolution in Eq. (5) can be evaluated efficiently in time domain if closed-form Green’s functions exist, such as in lossy media [3], the lack of robust and accurate time domain Green’s functions further complicates the matrix-fill step for layered media. The convolution in Eq. (5) suggests that $\mathbf{C}$ be computed in frequency domain:

$$
\hat{\mathbf{C}}(\mathbf{r},\mathbf{r}',\omega) = \hat{\mathbf{G}}^{\text{ref}}(\mathbf{r},\mathbf{r}',\omega) \hat{\omega} \hat{T}(\omega).
$$

(6)

Frequency domain Green’s functions for layered media are typically computed from Sommerfeld integrals [2]. Although asymptotic expansions and approximate closed forms are also used [7,8], numerical integration of Sommerfeld integrals on a revised path that does not include any singularities [8], while expensive, is the most robust, accurate, and general purpose method for computing frequency domain Green’s functions.
In this work, the Green’s functions $\hat{G}_\text{ref}(\mathbf{r}, \mathbf{r}', \omega)$ are computed by adaptive Simpson integration and the number of Sommerfeld integrals is analytically reduced similar to [2]. Then, the time domain functions $\hat{C}(\mathbf{r}, \mathbf{r}', (t-t')\Delta t)$ are obtained by computing $\hat{C}(\mathbf{r}, \mathbf{r}', \omega)$ as in Eq. (6) over a range of frequencies and using inverse FFTs. This, however, is not straightforward since the layered-media Green’s functions are typically neither time- nor band-limited. Nonetheless, the Green’s functions are convolved by currents that are typically excited by band-limited incident pulses during the MOT simulation and moreover the analysis is carried out for a finite number of time steps $N_t$. Hence, $\hat{C}(\mathbf{r}, \mathbf{r}', \omega)$ can be made band-limited before the inverse FFTs without affecting the MOT simulation. Various approaches have been used to limit the frequency range of Green’s functions: The frequency domain Green’s functions can be computed only up to the highest effective frequency of the incident pulse (in effect multiplying the frequency domain Green’s function with a rectangular pulse). This could, however, lead to excessive broadening of the time domain response and to time domain aliasing (due to the very slow decay of the sinc function). To minimize time domain aliasing, frequency domain Green’s functions should be windowed by smoother functions. Here, the approximate prolate spheroid functions [9] are used since they are not only band-limited but are also optimal windows in the sense that they are maximally short in time for a given time-bandwidth product. Even then, aliasing can occur due to the slow decay of time domain Green’s functions. This can be minimized by computing the inverse FFT at more than $N_t$ time steps and discarding the extra results.

Once the MOT matrices are computed as described, the solution of the MOT system of Eq. (3) has to be carried out. The solution is accelerated by the use of uniform meshes that lead to block-Toeplitz $Z_{l-l'}$ matrices. Notice that using uniform meshes also reduces the fill-time because the dyads $\hat{C}(\mathbf{r}, \mathbf{r}', (t-t')\Delta t)$ are computed only for $O(N_s)$ points.

3. Numerical Results

The proposed adaptive integration and inverse FFT (AI+IFFT) method for filling the MOT matrices is verified by comparing it with the Cagniard-de Hoop method [8]. The entries of $C_{y_2}(\mathbf{r}, \mathbf{r}', t)$ found by the two methods at sample times are plotted in Fig. 2 for $r'=(0,0,5 \text{ mm})$ and $r=(5 \text{ mm},3.5 \text{ mm},10 \text{ mm})$. Here $\varepsilon_{r_1}=1$, $\varepsilon_{r_2}=4$, $\Delta t=5/3 \text{ ps}$, and Eq. (6) is computed from 0 to 600 GHz at every 1.2 GHz (the window function smooths Green’s function for frequencies above 60 GHz). Good agreement is observed.

Next, the accuracy of the time domain simulations is verified by analyzing transient scattering from a $2 \text{ m} \times 2 \text{ m}$ square plate residing in free space $4 \text{ m}$ above the dielectric medium. The incident field is a $y$-polarized plane wave propagating in $-z$ direction. The time dependence of the incident wave is a baseband Gaussian with a bandwidth of 300 MHz. The plate is discretized using $N_s=1,984$ rooftop functions, the simulation is done for $N_t=1,024$ time steps, $\Delta t=5/24 \text{ ns}$, $\varepsilon_{r_1}=1$, and $\varepsilon_{r_2}=20$. Figure 3(a) compares the $y$-component of the current density near the center of the plate with the free-space simulation. Fig. 3(b) compares the Fourier transform of the $y$-polarized scattered field to fields obtained by a frequency-domain conjugate-gradient FFT (CG-FFT) solver along a line $2 \text{ m}$ above the plate at a number of frequencies.
References


Fig. 1 The dielectric half-space scenario

Fig. 2 Eq. (6) computed by two methods

Fig. 3 Transient scattering from a 2m x 2m plate: (a) Current density compared to free space simulations (ε_r2=1), (b) Scattered fields compared to frequency domain simulations.