Abstract: An efficient method for calculating the coupling of an exterior electromagnetic signal to a device on a printed circuit board inside of a cavity is considered for a wire penetrating an aperture in the cavity and making contact with the trace on the printed circuit board. Results are presented in both the frequency and time domains, and the case of a damped sinusoidal pulse is considered in some detail. It is shown analytically how the properties of the signal should be chosen to maximize the maximum induced voltage at the device, and this is confirmed by simulations.

INTRODUCTION

Many electronic systems are housed inside of metallic enclosures (cavities) that often have apertures. In many cases, wires or cables pass through one or more apertures and terminate inside the cavity, often ending on a printed circuit board (PCB). The system is illuminated by an incident wave, such as a plane wave. The voltage at the device port for this type of problem is calculated here in the frequency domain using an efficient "hybrid" method that separates the analysis of the PCB from that of the cavity [1]. This method, although approximate, accurately captures the different types of system resonances such as external cavity, wire, and internal cavity resonances.

The device port is assumed to be open-circuited (or terminated in a linear load). The system considered here is thus linear (at least up to the time of the device failure). With this assumption, the system response in the frequency domain can be used with the Fourier transform to find the time-domain response at the device port due to an incident time-domain electromagnetic plane wave. For the special case of a damped-sinusoidal plane-wave pulse, an analysis is given to show how the parameters of the pulse should be chosen to maximize the peak output voltage at the device port.

TIME DOMAIN ANALYSIS

A time-varying incident plane-wave pulse and the output voltage at the device port are related by a linear, time-invariant, continuous-time system. The system consists of the feed wire, the cavity, the PCB trace, and any linear loads that terminate portions of the PCB trace before it arrives at the device port of interest (which is assumed here to be open-circuited). Figure 1 shows a particular system used for the results. The PCB substrate is actually replaced by air in this figure for simplicity, corresponding to a transmission-line wire that is 1 mm above the bottom of the cavity. The time variation of the incident plane wave is assumed to be a damped sinusoidal pulse, having the form

\[ p(t) = Be^{-bt} \sin(\omega t)u(t), \]

where \( u(t) \) is the unit step function, \( \omega_c \) is the radian frequency of the carrier signal, and \( b \) is the decay parameter of the pulse. The parameter \( b \) is related to the quality \( (Q) \) factor of the pulse by \( b = -\omega_c/2Q_s \). The \( Q_s \) of the signal is inversely related to the bandwidth of the pulse. The relative bandwidth in the frequency domain is defined from the –10 dB bandwidth limits \( \omega^+ \) and \( \omega^- \), and is given approximately as \( BW_R = \Delta\omega / \omega_s = \left(\omega^+ - \omega^- \right) / \omega_s = 3 / Q_s \).
Results
Two different damped sinusoidal pulses are chosen, labeled as (1) or (2) in Fig. 2. The amplitude of each pulse, \( B \), is set to unity. Pulse (1) has a broad bandwidth (small \( Q_s \)) while pulse (2) has a narrow bandwidth (large \( Q_s \)) and a carrier frequency that is centered at one of the cavity mode resonances of the system (the 011 mode). The normalized Fourier transforms of the pulses are shown superimposed with the system response as a function of frequency in Fig. 2.

Pulse 1
Figure 3 shows the output response from the wide bandwidth pulse excitation (1). The pulse has a center frequency of 0.4 GHz. The signal quality factor is \( Q_s = 12.1 \) and the corresponding relative bandwidth is 0.25. The output signal is rather complicated, and exhibits interference between several cavity-mode resonances. In the late time, the interference is mainly a beating between two cavity resonances, and eventually (beyond the scale of the plot) a single cavity resonance response would dominate.

Pulse 2
Pulse 2 is centered about the (011) cavity-mode resonance of the system. The pulse has a center frequency at 0.51 GHz. Two different values of the signal quality factor \( Q_s \) are used. One value is greater than the \( Q_c \) value of this cavity mode and the other is smaller. The two values of \( Q_s \) are 510 and 51. The value of \( Q_c \) for this cavity mode is 128. Figure 4 shows the output response from these two narrow bandwidth pulses.

The late-time response of the output signal is observed to be dependent on the pulse quality factor \( Q_s \) (relative to \( Q_c \)). The envelope of the late-time response of the output signal has the same shape as the envelope of the cavity response when \( Q_s \) of the cavity is larger than \( Q_c \). In contrast, when \( Q_s \) is larger than \( Q_c \) of the cavity, the envelope of the late-time response has the same shape as the envelope of the input signal.

MAXIMIZATION OF THE PEAK OUTPUT RESPONSE

To approximate the output voltage, the system impulse response is first approximated assuming a single cavity resonance (i.e., a single-pole response) as

\[
h(t) = A e^{-at} \sin(\omega_c t) u(t),
\]

where \( \omega_c \) is the resonant frequency of the cavity mode. To maximize the output signal, the work of Baum [2] indicates that the carrier frequency of the pulse signal should be chosen to be the same as the center frequency of the resonant mode, \( \omega_c = \omega_c = \omega_c \).

The output voltage is then

\[
v(t) = \frac{AB}{2a_0} \left[ \frac{\sin(\omega_0 t)}{2} \left( e^{-at} + e^{-bt} \right) \cos(\omega_0 t) \left( e^{-at} - e^{-bt} \right) \right] \approx \frac{AB}{2} \left( \frac{e^{-at} - e^{-bt}}{a-b} \right) \cos(\omega_0 t).
\]

There are two types of constraints for the input pulse to be considered: amplitude constraint and energy constraint. Assuming that the amplitude of the input pulse is fixed (amplitude constraint), the envelope of the output signal reaches a maximum at some time, \( t_{\text{max}} \). The magnitude of the envelope function at time \( t_{\text{max}} \) is given by the function

\[
F_A(x) = f(t_{\text{max}}) = \frac{AB}{a} \frac{1-x}{x^{x-1}},
\]

where \( x = b/a \). A plot of (4) reveals that the function \( F_A \) increases indefinitely as \( x \) increases. Hence, the smaller \( b \) is chosen (i.e., the larger \( Q_s \) is made), the higher will be the peak output response.

If an energy constraint is imposed on the input pulse, the envelope function of the output voltage at time \( t_{\text{max}} \) is approximated by

\[
F_E(x) = \frac{2A}{\sqrt{a}} \frac{x^{\frac{1}{x^{x-1}}}}{\left( x^{\frac{1}{x^{x-1}}} \right)}.
\]

In this case, the function \( F_E \) is maximized when \( b = a \). Physically, this means that the quality factor \( Q_s \) of the signal is chosen equal to the quality factor \( Q_c \) of the cavity resonance.

Results
Three sinusoidal pulses with unit energy are applied to the system. The carrier frequencies of the pulses are chosen to be the same as the resonant frequency of the 110 cavity mode, which is 0.5755 GHz. The decay parameter \( a \) of this
cavity mode is 3.285E+06 Hz. The bandwidths of the pulses are selected so that the corresponding parameter \( x \) is equal to 0.25, 1, and 4. Figure 5 shows the results. The envelope of the output voltage is seen to be maximized when \( x = 1 \).

CONCLUSIONS

An efficient numerical scheme, denoted here as the “hybrid method,” can be used to determine the frequency-domain signal at the device port of a device on a printed-circuit board, due to an incident exterior electromagnetic wave. The printed circuit board is assumed to be inside a cavity, and a feed wire penetrates an aperture in the cavity to make contact with a trace on the board. Using the Fourier transform, time-domain signals at the device port due to time-domain waves impinging on the cavity may be determined. An analytical result shows that for a damped sinusoidal incident wave, the peak voltage level at the device port is maximized when the signal frequency and \( Q \) matches the resonance frequency and \( Q \) of a corresponding cavity mode; the result is confirmed by numerical calculations.

REFERENCES:


Figure 1. Structure used to obtain results. A feed wire penetrates an aperture in a conducting cavity and makes contact with a transmission-line wire. The aperture is square with dimensions 6\( \times \)6 cm and is centered horizontally on the left side of the box. The center of the aperture is 0.15 m above the bottom of the box. The feed wire has a radius \( a_2 = 0.25 \) mm, and protrudes a distance of 12 cm outside the box. The feed wire extends 20 cm inside the box before bending down to make contact with the transmission-line wire. The transmission-line wire has a radius \( a_1 = 0.383 \) mm and is 1 mm above the bottom of the box. The transmission-line wire has a total length of 26 cm, and extends 10 cm to the left to reach the open-circuit port, and 16 cm to the right to reach the 50 \( \Omega \) load. The incident plane wave is traveling vertically downward and is polarized with the electric field parallel to the feed wire [1].

Figure 2. Magnitude of the system response versus frequency. Also shown on the plot are sketches of the spectral content for the two different pulses that are used. Pulse 1 has a large bandwidth that spans several resonances. Pulse 2 is centered around a cavity mode (the 011 mode, with a resonance at about 0.51 GHz).
Figure 3. The open-circuit port voltage excited by a wide bandwidth input pulse (pulse (1) in Fig. 2). The wide bandwidth pulse results in a complicated ringing in the output response.

Figure 4. The open-circuit port voltage excited by a narrow bandwidth damped sinusoidal (pulse (2) in Fig. 2) with (a) $Q_s = 510$, (b) $Q_s = 51$. The high-$Q$ input pulse results in a much larger output response. Also shown are the normalized envelopes of the signal and the 011 cavity response.

Figure 5. Envelopes of the output signal for three different input pulses. The pulses are chosen to have unit energy and have the same carrier frequency as the resonant frequency of the 110 cavity-mode. The decay parameter of the pulse $(b)$ is related to the decay parameter of the cavity 110 mode $(a)$ by $x = a/b$. The peak output voltage is maximized when $x = 1$, as predicted by the theory. This corresponds to the input pulse having the same $Q$ as the cavity response.