Abstract—The scattering of a plane electromagnetic wave axially incident on the convex side of a multilayered paraboloidal structure is considered. An exact solution is obtained via geometrical optics, when the materials of all layers are isorefractive to one another. The solution is given explicitly for single- and double-layer radomes, and via a recursive algorithm for radomes with three or more layers. Previously known exact results (conducting paraboloid, isorefractive paraboloid, Fabry-Perot etalon) are particular cases of our general solution. Extensive numerical results are shown and discussed.

Index Terms—Electromagnetic scattering, isorefractive media, radomes.

I. INTRODUCTION

RADOMES are important components of antenna systems, both ground-based and airborne. Their electromagnetic behavior has been the subject of many investigations during the past half century, and these have led to a multitude of techniques for analyzing radomes behavior in the presence of a variety of sources. Since the complicated geometrical structure of radomes, that is often dictated by aerodynamic considerations, precludes an exact solution to the electromagnetic boundary-value problem, either asymptotic techniques or numerical methods have been employed. A comprehensive review of previous studies is beyond the scope of this paper; hence only some typical works are mentioned. From these and the reference therein, it is possible to acquire an overview of the state of the art in the understanding of electromagnetic behavior of radomes.

Techniques employed in the study of radomes may be divided into four broad categories: radomes idealized as shapes that can be studied exactly; use of simple geometries to model complex ones; asymptotic methods (ray tracing); numerical methods. Of course, approaches have been used that combine some of the above techniques. Among the shapes that can be handled exactly, planar, cylindrical and spherical radome structures have been studied extensively; worthy of special mention are the use of complex ray tracing in cylindrical radome shells [1], the analysis of a spherically-layered radome with two-dimensional aperture fields inside it [2], and the use of dyadic Green’s functions in multilayer spheroidal radomes [3].

Results from canonical problems involving planar and circular cylindrical structures have been used as the starting point to model more complex geometries. For example, planar slab results have been applied to transmission through a radome in studying boresight errors [4]. The exact Green’s function for a planar dielectric slab has provided as estimate of equivalent surface currents on a radome, and these currents have been used in the physical-optics calculation of the scattered field [5]. The Green’s function of a layered circular cylinder has been employed to account for the local curvature of a cylindrical radome of arbitrary section [6].

A copious literature exists on ray-tracing methods in the analysis of radomes, and this is understandable because the characteristic dimensions of a radome are typically large compared to the wavelength. We only mention the analysis of rays that are reflected on the internal surface of the radome and their dependency on the gimbal angle [7], [8], and the use of Poisson’s summation to handle the ray series arising from ray tracing in a two-dimensional multilayer radome [9].

Equally abundant are numerical studies of radomes, based on integral equations, or finite-element, or finite-difference, or hybrid techniques. A volume integral equation has been combined with a multilevel fast multipole algorithm [10], [11]. A surface integral equation has been used for a radome of arbitrary shape in the presence of an array of dipoles [12]. A finite element method with an absorbing boundary condition has been used to find the scattering by an axisymmetric radome, and then reciprocity has been invoked to find the radiation for an array of sources inside the radome [13]. A combination of the FEM and the periodic moment method has also been employed [14]. A curvilinear FDTD formulation has been developed to model thin curved dielectric sheets [15]. An interesting hybrid approach used the method of moment near the tip of the radome, where the radii of curvature may not be very large compared to the wavelength, and physical optics elsewhere [16].

From the preceding survey, it is evident that no exact solutions are available for radomes whose shape approaches a realistic one, especially for radomes on the nose of aircraft or missiles. The purpose of this work is to present such an exact solution for a radome whose surfaces are coaxial and confocal paraboloids of revolution, when the primary field is a plane wave axially incident on the convex side of the radome. The materials in the radome layers and in the surrounding space are isorefractive to one another, i.e. the refractive index (and consequently the wave number) is the same everywhere, but the intrinsic impedances of two neighboring materials have different values. All the exact solutions presented in this work coincide with the geometrical optics solutions. The analysis is conducted in the phasor domain, with the time-dependence.
factor $\exp(j\omega t)$ omitted throughout.

Our exact solutions are made possible by the unique behavior exhibited by the paraboloid of revolution. Schensted [17] proved that a PEC convex paraboloid under axial plane wave incidence has geometrical optics as the exact solution, and Lee [18] showed that such simple solution is not possible for other shapes. More recently, Roy and Uslenghi [19] proved that geometrical optics provides the exact solution when the paraboloid separates two isorefractive media; the previous result [17] for a PEC paraboloid follows as a particular case.

Some preliminary results, which did not include the explicit formulation for the double-layer radome and extensive numerical data, were previously presented at symposia [20], [21].

The geometry of the problem is discussed in section II, and the solutions for single-layer and double-layer radomes are given in sections III and IV, respectively. The case of a radome consisting of three or more layers is solved exactly in section V via a recursive algorithm. Numerical results are presented and discussed in section VI.

II. GEOMETRY OF THE PROBLEM

We consider paraboloidal coordinates $(\xi, \eta, \varphi)$ related to the rectangular Cartesian coordinates $(x, y, z)$ by the transformation:

$$
\begin{align*}
  x &= 2\sqrt{\xi\eta} \cos \varphi \\
  y &= 2\sqrt{\xi\eta} \sin \varphi \\
  z &= \xi - \eta
\end{align*}
$$

(1)

where $0 \leq \xi < \infty$, $0 \leq \eta < \infty$, and $0 \leq \varphi < 2\pi$.

![Fig. 1: Geometry of multi-layered paraboloidal radome](image_url)

The $z$-axis is the axis of symmetry and the $\xi = \text{constant}$ and $\eta = \text{constant}$ surfaces are paraboloids of revolution with foci at the origin $x = y = z = 0$, whereas the surfaces $\varphi = \text{constant}$ are semi-planes originating in the $z$-axis.

The surfaces that separate different isorefractive media are paraboloids of revolution $\eta = \text{constant}$, and the axially incident primary plane wave propagates in the positive $z$-direction with wavenumber $k$ and, without loss of generality, is assumed to be polarized in the $x$-direction, so that its electric field $\mathbf{E}^i$ and magnetic field $\mathbf{H}^i$ are given by the column vector:

$$
\left( \begin{array}{c}
  \mathbf{E}^i \\
  \mathbf{H}^i
\end{array} \right) = \left( \begin{array}{c}
  \hat{x} \\
  Y_1 \hat{y}
\end{array} \right) e^{-jk(\xi-\eta)}
$$

(2)

where $\hat{x}$ and $\hat{y}$ are the unit vectors oriented in the positive $x$ and $y$ directions, respectively, whereas $Y_1$ is the intrinsic admittance of medium $1$, which is linear and isotropic, and fills the space on the convex side of the structure.

The structure considered herein consist of one or more paraboloids of revolution $\eta = \eta_1, \eta_2, \ldots (\eta_1 > \eta_2 \ldots)$ that separate regions of space filled with linear and isotropic materials that are isorefractive to one another. Medium $l$ occupies the space between the paraboloids $\eta = \eta_{l-1}$ and $\eta = \eta_l$, has electric permittivity $\varepsilon_l$ and magnetic permeability $\mu_l$, and is characterized by a wavenumber $k$ and all intrinsic impedance $Z_l$ given by

$$
k = \omega \sqrt{\varepsilon_l \mu_l}, \quad Z_l = Y_{l-1} = \sqrt{\mu_l/\varepsilon_l}
$$

(3)

where $k$ is the same for all media. In particular, the last medium encountered as one proceeds in the positive $z$-direction may be a perfect electric conductor (PEC), in which case its intrinsic impedance is zero. Thus, the PEC paraboloid covered by isorefractive layers is a particular case of a multi-layer paraboloidal radome.

When an electromagnetic field is incident from medium $l$ on the surface $\eta = \eta_l$ separating it from medium $l+1$, the reflection and transmission coefficients $R_{l,l+1}$ and $T_{l,l+1}$ of the electric field are given by:

$$
R_{l,l+1} = \frac{1 - \zeta_{l,l+1}}{1 + \zeta_{l,l+1}}, \quad T_{l,l+1} = \frac{2}{1 + \zeta_{l,l+1}} = 1 + R_{l,l+1}
$$

(4)

where

$$
\zeta_{l,l+1} = Z_l/Z_{l+1}.
$$

(5)

Note that these coefficients are independent of both polarization and angle of incidence of the incident field; also, the angle of refraction equals the angle of incidence (these properties are peculiar to isorefractive media [22]).

In order to simplify the analysis of the following sections, it is expedient to introduce the basic vectors:

$$
\mathbf{e}_o = (1 - \frac{\xi}{\eta} \cos 2\varphi)\hat{x} - \frac{\xi}{\eta} \sin 2\varphi \hat{y} + 2 \sqrt{\frac{\xi}{\eta} \cos \varphi} \hat{z},
$$

(6)

$$
\mathbf{h}_o = \frac{\xi}{\eta} \sin 2\varphi \hat{x} - (1 + \frac{\xi}{\eta} \cos 2\varphi) \hat{y} - 2 \sqrt{\frac{\xi}{\eta} \sin \varphi} \hat{z}. \quad (7)
$$

Note that:

$$
|\mathbf{e}_o| = |\mathbf{h}_o| = 1 + \frac{\xi}{\eta},
$$

(8)

$$
\mathbf{e}_o \cdot \mathbf{h}_o = 0. \quad (9)
$$

$$
\mathbf{e}_o \times \mathbf{h}_o = (1 + \frac{\xi}{\eta}) \left[ 2 \sqrt{\frac{\xi}{\eta} \rho} + (\frac{\xi}{\eta} - 1) \hat{z} \right]. \quad (10)
$$

where $\rho = \hat{x} \cos \varphi + \hat{y} \sin \varphi$ is the radial unit vector in circular cylindrical coordinates.
III. SINGLE-LAYER STRUCTURE

A single-layer structure consists of two coaxial and confocal paraboloids of revolution defined by surfaces \( \eta = \eta_1 \) and \( \eta = \eta_2 < \eta_1 \). The medium 1 occupying the space \( \eta > \eta_1 \) on the convex side of the radome and the medium 3 occupying the space \( \eta < \eta_2 \) on the concave side of the radome are separated by the radome layer \( \eta_2 < \eta < \eta_1 \), as shown in Fig. 2. The three media are isorefractive to one another.

Consider a point \( P \) in medium 1, and let us calculate the total field at \( P \) according to geometrical optics. In addition to the incident field, there exists an infinite series of reflected rays that appear to be emanating from the focus \( F \). The fields associated with the rays propagating in the total geometrical optics field (described in terms of the basic vector \( \mathbf{E}_i, \mathbf{H}_i \)) are separated from the other representing rays propagating from the focus \( F \); each series may be summed in closed form if the media are passive, yielding the total field \( (\mathbf{E}_2, \mathbf{H}_2) \) inside the layer:

\[
\begin{align*}
(\mathbf{E}_2, \mathbf{H}_2) &= \left( \begin{array}{c} \hat{x} \\ Y_2 \hat{y} \end{array} \right) \frac{\eta_1(1 + R_{12})e^{-jk(\xi - \eta)}}{\eta_1 + \eta_2 R_{12} R_{23} e^{-2jk(\eta_1 - \eta_2)}} \\
&\quad + \left( \begin{array}{c} e_o \\ Y_2 h_o \end{array} \right) \frac{e^{-jk(\xi + \eta - 2\eta_1)}}{\xi + \eta} \\
&\quad \times \frac{\eta_1 \eta_2 R_{23} (1 + R_{12}) e^{-2jk(\eta_1 - \eta_2)}}{\eta_1 + \eta_2 R_{12} R_{23} e^{-2jk(\eta_1 - \eta_2)}} .
\end{align*}
\]

(13)

Finally, the total field \( (\mathbf{E}_3, \mathbf{H}_3) \) on the concave side of the radome, i.e. for \( \eta < \eta_2 \), may be similarly summed to yield:

\[
\begin{align*}
(\mathbf{E}_3, \mathbf{H}_3) &= \left( \begin{array}{c} \hat{x} \\ Y_3 \hat{y} \end{array} \right) \frac{\eta_1(1 + R_{12}) (1 + R_{23}) e^{-jk(\xi - \eta)}}{\eta_1 + \eta_2 R_{12} R_{23} e^{-2jk(\eta_1 - \eta_2)}} .
\end{align*}
\]

(14)

If the geometrically-derived fields (11-14) are inserted into Maxwell’s equations, it is found that these equations are satisfied exactly. These fields also exactly satisfy the boundary conditions at \( \eta = \eta_{2,3} \). Thus, because of the uniqueness theorem, it follows that our geometrical optics solution is the exact solution to the scattering problem depicted in Fig. 2.

Several structures may be considered as particular cases of our result. First, the isorefractive paraboloid studied in [19] follows from our result by letting \( Z_2 = Z_3 \), implying that \( R_{23} = 0 \); if in addition \( Z_2 = 0 \), the PEC paraboloid of [17] also follows. A limit when the focal distance is allowed to tend to infinity while \( \eta_1 - \eta_2 \) is kept constant yields the known result for the Fabry-Perot etalon, or interferometer.

The backscattering radar cross section is obtained from (12):

\[
\sigma = 4\pi \eta_1^2 \left( \frac{R_{12} + \frac{\eta_2}{\eta_1} R_{23} e^{2jk(\eta_1 - \eta_2)}}{1 + \frac{\eta_2}{\eta_1} R_{12} R_{23} e^{2jk(\eta_1 - \eta_2)}} \right)^2 .
\]

(15)

The particular case of medium 1 identical to medium 3 is obtained by letting \( R_{23} = R_{21} = -R_{12} \) in (15), whereas the case of a coated PEC paraboloid \( (Z_3 = 0) \) is obtained by letting \( R_{23} = -1 \) in (15).

IV. DOUBLE-LAYER STRUCTURE

A double-layer radome consists of three coaxial and confocal paraboloidal surfaces \( \eta = \eta_{1,2,3} \) (\( \eta_1 > \eta_2 > \eta_3 \)) separating four media isorefractive to one another. The structure is a particular case of that shown in Fig. 1, and the analysis proceeds in a manner similar to the single-layer structure. In each medium, the total field consists of two infinite series of ray-optical contributions. The first series accounts for all rays parallel to the \( z \)-axis and may be summed in closed form if the media are passive, leading to the field:

\[
\begin{align*}
(\mathbf{E}_l^{in}, \mathbf{H}_l^{in}) &= A_l \left( \begin{array}{c} \hat{x} \\ Y_l \hat{y} \end{array} \right) e^{-jk(\xi - \eta)}, \quad (l = 1, 2, 3, 4)
\end{align*}
\]

(16)

where \( A_l \) are scalar coefficients. The second series accounts for all rays that appear to be emanating from the focus \( F \), and
may be similarly summed in closed form, yielding the field:

\[
\begin{pmatrix}
E_l^{\text{out}} \\
H_l^{\text{out}}
\end{pmatrix}
= B_l \begin{pmatrix}
e_o \\
y_l h_o
\end{pmatrix} \frac{e^{-jk(\xi + \eta)}}{\xi + \eta}, \quad (l = 1, 2, 3, 4)
\]

(17)

where \(B_l\) are scalar coefficients. The total field is given by the sum of the field (16) and (17):

\[
\begin{pmatrix}
E_l \\
H_l
\end{pmatrix}
= A_l \begin{pmatrix}
\tilde{x} \\
y_l \tilde{y}
\end{pmatrix} e^{-jk(\xi - \eta)} + B_l \begin{pmatrix}
e_o \\
y_l h_o
\end{pmatrix} \frac{e^{-jk(\xi + \eta)}}{\xi + \eta}.
\]

(18)

The coefficients \(A_1\) and \(B_1\) in medium 1 (\(\eta > \eta_1\)) on the convex side of the structure are:

\[
A_1 = 1, \quad \text{(incident field),}
\]

\[
B_1 = C[\eta_1^2 \eta_2 R_{12} e^{2k\eta_1} + \eta_2^2 R_{23} e^{2k\eta_2} + \eta_2 \eta_3 R_{34} e^{2k\eta_3} + \eta_1 \eta_3 R_{12} R_{23} R_{34} e^{2k(\eta_1 - \eta_2 + \eta_3)}],
\]

(20)

where

\[
C = (\eta_1 \eta_2 + \eta_1 \eta_3 R_{23} R_{34} e^{2k(\eta_3 - \eta_2)} + \eta_2^2 R_{12} R_{23} e^{2k(\eta_2 - \eta_1)} + \eta_2 \eta_3 R_{12} R_{34} e^{2k(\eta_3 - \eta_1)})^{-1}.
\]

(21)

In medium 2 (\(\eta_1 > \eta > \eta_2\)) occupying the outer layer of the radome:

\[
A_2 = C\eta_1(1 + R_{12})(\eta_2 + \eta_3 R_{23} R_{34} e^{2k(\eta_3 - \eta_2)}),
\]

\[
B_2 = A_2 \frac{\eta_2 R_{23} e^{2k\eta_2} + \eta_3 R_{34} e^{2k\eta_3}}{1 + \frac{\eta_2}{\eta_3} R_{23} R_{34} e^{2k(\eta_3 - \eta_2)}}.
\]

(22)

In medium 3 (\(\eta_2 > \eta > \eta_3\)) occupying the inner layer of the radome:

\[
A_3 = A_2 \frac{1 + R_{23}}{1 + \frac{\eta_2}{\eta_3} R_{23} R_{34} e^{2k(\eta_3 - \eta_2)}},
\]

\[
B_3 = A_3 \eta_3 R_{34} e^{2k\eta_3}.
\]

(24)

(25)

Finally, in medium 4 (\(\eta < \eta_3\)) on the concave side of the structure:

\[
A_4 = A_3(1 + R_{34}),
\]

\[
B_4 = 0.
\]

(26)

(27)

The above solution for the double-layer radome satisfies both Maxwell’s equations and the boundary conditions exactly. Hence, this geometrical optics solution is the exact solution of the boundary-value problem. Several particular cases, such as the PEC paraboloid coated by a double layer of isorefractive materials, easily follow.

The backscattering radar cross section of the double-layer structure is exactly given by:

\[
\sigma = 4\pi |B_1|^2.
\]

(28)

V. MULTI-LAYER STRUCTURE

The derivation of the previous two sections may be extended to the case of a paraboloidal radome consisting of more than two layers, yielding an exact, geometrical optics solution. However, the explicit formulas become very cumbersome and it is therefore expedient to formulate the problem in terms of a recursive algorithm. The total field in medium \(l\) is written in the form (see Fig. 1):

\[
\begin{pmatrix}
E_l \\
H_l
\end{pmatrix}
= T_{l,1} \begin{pmatrix}
\tilde{x} \\
y_l \tilde{y}
\end{pmatrix} e^{-jk(\xi - \eta)} + \tilde{R}_{l,l+1} T_{l,1} \begin{pmatrix}
e_o \\
y_l h_o
\end{pmatrix} \frac{e^{-jk(\xi + \eta)}}{\xi + \eta}.
\]

(29)

where the coefficients obey the recursive relations:

\[
\tilde{R}_{l,l+1} = \eta_1(\eta_l R_{l,l+1} e^{2k\eta_l} + \tilde{R}_{l+1,l+2}),
\]

\[
T_{l,l+1} = \eta_1(1 + R_{l,l+1}) e^{2k\eta_l} R_{l+1,l+2},
\]

\[
T_{l,1} = T_{l,2} T_{2,3} \ldots T_{l-1,1} \prod_{m=1}^{l-1} \tilde{T}_{m,m+1}
\]

(30)

(31)

(32)

and the initial conditions:

\[
T_{l,1} = 1,
\]

\[
\tilde{R}_{n+1,n+2} = 0,
\]

\[
T_{n+1,n+1} = 1 + R_{n,n+1},
\]

\[
\tilde{R}_{n,n+1} = \eta_n R_{n,n+1} e^{2k\eta_n}.
\]

(33)

(34)

(35)

(36)

The solution (29-36) is exact. In particular, the exact backscattering radar cross section is given by

\[
\sigma = 4\pi |\tilde{R}_{1,2}|^2.
\]

(37)

VI. NUMERICAL RESULTS

The backscattering radar cross section (BRCS) has been calculated for several radome structures, on the basis of the formulas obtained in the previous section. All results have been normalized to the value \(4\pi \eta_1^2\), which is the BRCS of a structure whose outer surface \(\eta = \eta_1\) is either PEC (\(Z_2 = 0, R_{12} = -1\)), or a perfect magnetic conductor (\(Y_2 = 0, R_{12} = 1\)):

\[
\sigma_N = \sigma/(4\pi \eta_1^2).
\]

(38)

The value of \(\sigma_N\) for a single isorefractive layer coating a PEC paraboloid is shown in Fig. 3 for \(\eta_1/\eta_2 = 1.5\) and four different frequencies, as a function of the ratio \(Z_2/Z_1\) between the intrinsic impedances of the layer and of the surrounding space. The BRCS is exactly zero for \(Z_2/Z_1 = 0.2\) when \(\eta_1/\lambda = 0.75\) and for \(Z_2/Z_1 = 5\) when \(\eta_1/\lambda = 1.5\), \(\lambda\) being the wavelength. More generally, the BRCS of a PEC paraboloid with a single coating is zero whenever \(\lambda = 4(\eta_1 - \eta_2)/m\) with \(m\) any positive integer, provided that:

\[
Z_2/Z_1 = \left(\frac{\eta_1}{\eta_2} + (-1)^m\right) / \left(\frac{\eta_1}{\eta_2} - (-1)^m\right).
\]

(39)
The value of $\sigma_N$ for a single isorefractive radome separating two regions of space occupied by the same medium ($Z_1 = Z_3, R_{23} = R_{32} = -R_{12}$) is shown in Fig. 4 for $\eta_1/\eta_2 = 1.5$ and four different frequencies, as a function of $Z_2/Z_1$. Note the $\sigma_N = 0$ only when $Z_2 = Z_1$, i.e. in the absence of a radome.

In general, the BRCS of any single-layer structure is zero when $\lambda = 4(\eta_1 - \eta_2)/m$ and

$$\frac{R_{12}}{R_{23}} = \eta_2(-1)^{m+1}$$

(40)

where $m$ is any positive integer. Also, the BRCS of such structure is periodic in frequency with period $c/[2(\eta_1 - \eta_2)]$, where $c$ is the velocity of light.

The normalized BRCS of a double-layer structure is shown in Fig. 5 when the medium on the concave side is a PEC ($Z_4 = 0$) and in Fig. 6 when the media on the convex and concave sides are the same ($Z_1 = Z_4$). In both cases, $\sigma_N$ is plotted for three frequencies as a function of $Z_2/Z_1$ when $Z_2/Z_1 = 5$ and $\eta_1/\eta_2 = 7/6, \eta_2/\eta_3 = 1.5$. The BRCS of any double-layer structure is periodic in frequency, with a period equal to the least common multiple among $c/[2(\eta_1 - \eta_2)], c/[2(\eta_1 - \eta_3)], c/[2(\eta_2 - \eta_3)]$.

Polar plots of the magnitude of the total electric and magnetic fields when the spaces on the convex and concave sides of the structure are the same medium (air) are shown in Figs. 7 and 8 for a single-layer radome and in Figs. 9 and 10 for a double-layer radome. With reference to spherical polar coordinates $(r, \theta, \phi)$ related to the rectangular coordinates $(x, y, z)$ of Fig. 1 in the usual manner, the field magnitudes are shown in the E-plane (\(\phi = 0\)) and H-plane (\(\phi = \pi/2\)) as function of $\theta$ along circles of radius $r = 5\eta_2$ for the single-layer radome and $r = 5\eta_3$ for the double-layer radome. The incident electric field is assumed to have a magnitude of $1V/m$, and the magnitudes of the total fields are in $V/m$ in
Fig. 7: Magnitude of electric field vs. $\theta$ for a single-layer radome with air backing ($R_{23} = R_{21} = -R_{12}$, $\eta_1 : \eta_2 : \lambda = 3 : 2 : 3$ and $Z_1 : Z_2 = 1 : 5$). The electric field is sampled on a circle with radius $r = 5\eta_2$ at E-plane and H-plane.

Fig. 8: Magnitude of magnetic field vs. $\theta$ for a single-layer radome with air backing ($R_{23} = R_{21} = -R_{12}$, $\eta_1 : \eta_2 : \lambda = 3 : 2 : 3$ and $Z_1 : Z_2 = 1 : 5$). The magnetic field is sampled on a circle with radius $r = 5\eta_2$ at E-plane and H-plane.

Fig. 9: Magnitude of electric field vs. $\theta$ for a double-layer radome with air backing ($R_{34} = R_{21} = -R_{12}$, $\eta_1 : \eta_2 : \eta_3 : \lambda = 4 : 3 : 2 : 3$ and $Z_1 : Z_2 : Z_3 = 1 : 3 : 5$). The electric field is sampled on a circle with radius $r = 5\eta_3$ at E-plane and H-plane.

Fig. 10: Magnitude of magnetic field vs. $\theta$ for a double-layer radome with air backing ($R_{34} = R_{21} = -R_{12}$, $\eta_1 : \eta_2 : \eta_3 : \lambda = 4 : 3 : 2 : 3$ and $Z_1 : Z_2 : Z_3 = 1 : 3 : 5$). The magnetic field is sampled on a circle with radius $r = 5\eta_3$ at E-plane and H-plane.

Figs. 7 and 9, and in $A/m$ in Figs. 8 and 10. As expected, the field magnitudes are constant at all points on the concave side the radomes.

The total electric and magnetic field magnitude are shown in the E- and H- planes for a PEC paraboloid coated by a double layer, in Figs. 11-14. The grey-scale plots cover a rectangular window centered at the focus $F$ of the structure and with dimension $20\lambda$ in both the $z$ direction and the direction perpendicular to the $z$ axis. Electric fields are in $V/m$ and magnetic fields in $A/m$, for an incident electric field of $1V/m$. 
VII. CONCLUSION

The exact solution has been obtained in closed form for the scattering of a plane wave axially incident on a radome structure consisting of any number of layers of different materials isorefractive to one another, when all surfaces separating different media are coaxial and confocal paraboloids of revolution. This exact solution coincides with the geometrical optics solution. Several numerical results were presented and discussed.

Our solution is important for two reasons. First, it enriches the limited list of canonical solutions that are presently known...
for penetrable structures. Second, it provides a validation tool for approximate analytical solutions and numerical solutions developed for the study of radiation and scattering by radome structures.

It may be possible to extend the scope of the results obtained herein by considering media that are almost isorefractive to one another and/or shapes that are almost paraboloidal, and employing a perturbation technique. The resulting analysis, however, appears to be quite cumbersome and would have to be performed in future investigations.

REFERENCES


