A Hybrid MoM-SPICE Technique for Field Coupling Analysis of Transmission Lines in Presence of Complex Structures

Yakup Bayram, Student Member, IEEE, and John L. Volakis, Fellow, IEEE

Abstract—A computationally efficient method for field coupling on multiconductor transmission lines near complex structures is proposed. Key to the method is the decomposition of the current on each transmission line into push-push and push-pull mode currents. The former accounts for the interactions between the transmission line bundle and the surrounding structure, whereas the latter is perturbation current accounting for the interactions among the wires within the bundle. The push-pull mode current is computed via the traditional transmission line approach by taking one of the wires in the bundle as the return/reference conductor. On the other hand, the push-push mode current is found by solving a test wire located along the reference transmission line in the presence of the surrounding structure. For this analysis, the surrounding structure is modeled via the Method of Moments and a SPICE-like simulator is used to simulate an equivalent coupling circuit model of the transmission lines extracted via the Partial Element Equivalent Circuit (PEEC) Method.

Several validation examples (including transmission lines inside automobile) are presented and it is also shown that the traditional Transmission line theory based on quasi-static analysis fails with increasing complexity of the surrounding structure.

I. INTRODUCTION

Electromagnetic field coupling analysis of multiconductor transmission lines near complex structures has been the subject of many extensive studies for the EMI/EMC evaluation of electronic systems in complex platforms such as automobiles, trains and airplanes. Of primary concern is coupling onto electronic modules through the harness illuminated by intentional or unintentional EM fields. Namely, of interest is the RF coupling onto cable bundles rather than directly onto electronic modules since the highest wavelength of interest is in the range of tenth of meters.

The primary approach taken in industry is to carry out extensive experiments on test body prototypes before mass production. However, experimental studies lead to inevitable cost as well as to increased time on performance evaluation. Therefore, theoretical predictions of field coupling onto multiconductor transmission lines is of importance. However, the geometrical complexities involved in the cable bundles make it intractable for full wave solvers. Hence, the Multiconductor Transmission Line Theory (MTLT) is typically preferred for field coupling analysis [1], [2], [3], [4], [5].

The advantage of MTLT for field coupling analysis is the ability to treat the test body and the bundle separately. In other words, the incident field on the transmission lines is computed in the absence of the cable bundles whereas the structure is analyzed with full wave solvers. Subsequently, the computed incident fields (scattered fields from the structure) are used as additional voltage and current sources within the MTLT formulation.

It should be noted that MTLT is limited to the cases where quasi-static conditions are met. Namely, it yields fairly good results for low frequency analysis [6], [7]. MTLT also does not account for self-radiation of the transmission lines. That is, MTLT only predicts the differential mode current, but for complex structures, the common mode current is required to better characterize the total current induced on the transmission lines. To overcome this limitation, a technique that modifies telegrapher’s coupling equations with additional iterative terms to account for non-static contributions was proposed in [8] for a finite single transmission line over a perfectly conducting infinite plane. Alternatively, Haase [9] proposed an iterative transmission line method, employing the full wave form of Telegrapher’s equations. Nevertheless, Haase’s validation was done for non-uniform multiconductor transmission lines over a perfectly conducting flat plate.

The typical challenge for practical EMI/EMC analysis is to model geometrical details. To address this, hybrid techniques (treating the complex structures with numerical methods and modeling the transmission lines with MTLT) were proposed in [7], [10], [11], [12], [13], [14], [15]. In [10], an iterative approach was followed to account for the multi-interactions between Printed Circuit Board (PCB) interconnects and the surrounding structure. However, the formulation in [10] is based on the iterative solution of the quasi-static MTLT coupling equations. Since the quasi-static conditions are well justified for PCBs, the standard MTLT method is adequate for the quasi-static cases.

To overcome the quasi-static restrictions posed by MTLT, the authors proposed modified Agrawal’s coupling equations (see [2]) with additional iterative voltage and current sources to account for non-static contributions caused by interactions between transmission line bundle and the surrounding structure [15]. The proposed method, referred to as TICE (Telegrapher’s Iterative Coupling Equations), was based on the field refinement technique to account for interactions in an iterative fashion, giving rise to additional iterative voltage and current sources. TICE was also validated for various geometries including transmission lines in cavity-like structures and troughs [15]. Nevertheless, TICE requires more CPU time due to the full wave analysis of the surrounding structure for every iteration. Further, it requires approximately ten times more dense discretisation in SPICE modeling than
transmission line theory to accurately account for interactions. Therefore, the implementation of TICE for entire automobile and aircraft harnesses becomes impractical, particularly when one considers the total length of the harness (on the order of kilometers) and the also large electrical size of the surrounding structure.

Large portion of the automotive or aircraft harness consists of mass wires which can be simply treated as scatterers (“mass wires” refer to wires which are not terminated or terminations of which can be neglected in the analysis). In other words, one can split the harness into portion of mass wires and wire sections containing individual wires with terminations. The latter can be treated with TICE for rigorous analysis. However, use of TICE for mass wires would be computationally unappealing due to their large number and high discretisation requirements. To tackle this issue, we propose a new approach employing a current decomposition method for mass transmission line bundles near the complex structures.

Under the proposed method, the current on each transmission line is decomposed into push-push (average) and push-pull (perturbation) mode currents (see Fig. 1). A similar convention was used by Schelkunoff and Friis [16] for the analysis of two wire antennas in free space. The former accounts for the interactions between the surrounding structure and the transmission lines, and the latter is responsible for the current perturbation due to interactions among the transmission lines forming the bundle. The push-pull mode current is computed via the traditional transmission line method by taking one of the transmission lines in the bundle, preferably the one at the center, as the return/reference conductor. To evaluate the push-pull mode current, we compute the current of a test wire located along the reference transmission line in the presence of the surrounding structure. SPICE models are then employed for the push-pull mode current to analyze the equivalent coupling circuit models extracted via PEEC (see [17], [18] and [19]).

In the subsequent sections, we first demonstrate the mathematical existence of push-push and push-pull mode currents. Then, we revisit TICE to derive the Telegrapher’s Generalized Coupling Equations (TGCE) which is eventually used to obtain the push-push mode current. The procedure employed to derive and compute the push-push and push-pull mode currents is described in sections IV and V, respectively. This is followed by analysis tools and several validation examples. Finally, in the discussion and conclusion sections, comments on the proposed method are given.

II. CURRENT DECOMPOSITION METHOD

In this section, we demonstrate that currents on any transmission line bundle can be decomposed into push-push and push-pull mode currents. Fig. 2 (showing a cable bundle) illustrates the proposed decomposition into push-push (denoted as $I_{pp}$) and push-pull (denoted as $I_{pl}$) modes. We choose one of the conductors to carry the return current as denoted by subscript “1”. Thus, the total push-pull mode current on the return conductor is equal to the sum of the push-pull mode current for each individual transmission line. This decomposition parallels the standard differential and common mode decomposition. However, the proposed push-push and push-pull modes are different from the common mode and differential mode currents. The differential and common mode currents are computed with the surrounding structure being the return conductor. However, for our decomposition, the return conductor can be chosen arbitrarily implying that different current values for the push-push and push-pull mode modes can be obtained for each wire depending on the choice of reference conductor.

Because of the possible non-physical decomposition associated with the push-push and push-pull mode, it is essential that currents on any transmission line can be represented by push-push and push-pull components (as depicted in Fig. 3). In Fig. 3, the left column vector is the total current induced on each transmission line ($N$ being the total number of transmission lines) whereas the column vector on the right hand side represents the push-push (average) and push-pull mode currents associated with each transmission line. As seen, the first matrix column is “ones” since the push-push mode current is the same on each transmission line. The remaining “ones” on each row refer to the push-pull mode current (perturbation current) associated with each transmission line. The negative “ones” on the last row of the composition matrix imply that the $1^{st}$ conductor is the return conductor.

To prove the mathematical existence of the decomposition in Fig. 3, it must be shown that the composition matrix is nonsingular. To do so, we proceed to show that the compo-
The determinant of the composition matrix \((CM_N)\) has a non-zero determinant. Looking at the determinant of \((CM_N)\), we have

\[
det(CM_N) = (-1)^{(N-1)} \det(CM_{N-1}) + \det(P_{N-1}) = (-1)^{(N-1)} \det(CM_{N-1}) + (-1)^{(N-3)(N-2)/2} + 1
\]

The \(CM_{N-1}\) and \(P_{N-1}\) are cofactor matrices displayed in Fig. 3 with solid and dash lines, respectively. It is easy to see that \(det(CM_2) = -2\) and eventually it follows that \(det(CM_N)\) is always non-zero. Therefore, the proposed \(I_{pp}\) and \(I_{pl}\) are unique for the chosen return conductor and reversible even though the return conductor is chosen arbitrarily.

### III. Telegrapher’s Generalized Coupling Equations (TGCE)

We now revisit TICE [15] to generalize them for the analysis of the push-push mode current. The primary goal in this section is to derive a new set of equations based on TICE and to establish a relation between the test wire and push-push mode current. To do so, we begin with TICE for an N conductor transmission line bundle surrounded by a complex structure (see Fig. 4).

\[
\begin{align*}
\frac{d[V_{ref}]}{dx} + j \omega [\bar{L}] [I] &= \left[ E_{exc}^{ref} \right] - j \omega [A]_{n-1} + j \omega [\bar{L}] [I]_{n-1} \\
\frac{d[I]}{dx} + j \omega [\bar{C}] [V_{ref}] &= \frac{d[I]}{dx} + j \omega [\bar{C}] [\phi]_{n-1}
\end{align*}
\]

where \(n\) is the iteration number. \(E_{exc}^{ref}\) is the total incident field in the absence of the transmission lines. At every iteration, vector and scalar potentials \([A]_{n-1}\) and \([\phi]_{n-1}\) are computed from the potential integrals:

\[
A_{n-1}(x_i, y_i, z_i) = \frac{\mu}{4\pi} \sum_{u=1}^{N} \int_{L_u}^{L_{n-1}} I_{n-1}(x, y, z) G(x, y, z; x, y, z) dx
\]

\[
\phi_{n-1}(x_i, y_i, z_i) = \nabla \cdot \bar{A}_{n-1}(x_i, y_i, z_i) - j \omega \mu e
\]

where \(G(x, y, z; x, y, z)\) is the Green’s function in the presence of the structure. Alternatively, \(A_{n-1}(x_i, y_i, z_i)\) and \(\phi_{n-1}(x_i, y_i, z_i)\) can be evaluated numerically with \(I_{n-1}(x, y, z)\) as the excitation.

Convergence of (2) is reached when \(I = I_n \approx I_{n-1}\). In other words, convergence occurs when the tangential electric field boundary condition and continuity equation are satisfied along the transmission lines. Thus, (2) reduces to

\[
\begin{align*}
\frac{d[V_{ref}]}{dx} + j \omega [\bar{L}] [I] &= V_f \\
\frac{d[I]}{dx} + j \omega [\bar{C}] [V_{ref}] &= I_f
\end{align*}
\]

with

\[
\begin{align*}
V_f &= [E_{exc}^{ref}] - j \omega [A] + j \omega [\bar{L}] [I_{induced}] \\
I_f &= \frac{d[I_{induced}]}{dx} + j \omega [\bar{C}] [\phi]
\end{align*}
\]

where \(I_{induced}\) is the total induced current on the transmission lines and \(A\) and \(\phi\) are the vector and scalar potentials respectively. The relations in (4) and (5) can be referred to as the Telegrapher’s Generalized Coupling Equations (TGCE). They imply that the equivalent transmission line model can be obtained through knowledge of the total current induced on the transmission lines in presence of the surrounding structure. This can be construed as an inverse problem such that the equivalent distributed voltage and current sources are computed from a knowledge of the total induced current. Thus, in our analysis, the total current induced on the test wire analysis can be used to derive the equivalent voltage and current sources which account for the interactions between the structure and the test wire.

In the next section, the TGCE is used to establish a relation between the push-push mode current and a single test wire current solved in the presence of the surrounding structure.

### IV. Push-Push Mode Current

As mentioned in the previous sections, the push-push mode current dominates the interactions between the transmission line bundle and the surrounding structure since the sum of the push-pull mode currents are zero within the bundle. To account for the interactions between the wire bundle and the surrounding structure, one can solve for a single test wire via a full wave analysis as shown in Fig. 5. It is best to place the test wire at the location of the return/reference conductor since it carries all the push-pull mode information.
Fig. 5. Test wire analysis: Original configuration (left) and test wire configuration (right)

As of our starting point, we begin with the TGCE associated with the test wire,

\[
\frac{dV_{\text{ref}_\text{test}}}{dx} + j\omega L_{\text{test}_1}I_{\text{test}} = V_{f_{\text{test}}}
\]

with

\[
V_{f_{\text{test}}} = E_{\text{exc},\text{test}}^{\text{exc}} - j\omega A_1 + j\omega (L\dot{I})_{\text{induced}}^{\text{induced}}
\]

where

\[
(L\dot{I})_{\text{induced}}^{\text{induced}} = L_1^1 I_{\text{induced}}^1 + \ldots + L_N^N I_{\text{induced}}^N
\]

and \(E_{\text{exc}}^{\text{exc}}\) is the incident electric field, \(A_1\) is the vector potential.

Upon using the relations in Fig. 3, we can rewrite (7) as

\[
\frac{dV_{\text{ref}_\text{test}}}{dx} + j\omega \sum_{k=1}^{N} L_{1k}I_{\text{pp}} + j\omega \sum_{k=2}^{N} (L_{1k} - L_{11})I_{\text{pl}_k} = V_{f_{\text{test}}}
\]

where \(E_{\text{exc},\text{test}}^{\text{exc}} = E_{1x}^{\text{exc}}\) and \(A_{\text{test}} = A_1\). Hence, \(V_{\text{ref}_\text{test}} = V_{\text{ref}_\text{f1}}\) (see [15]).

It was already shown in (4) that the induced current on the test wire \((I_{\text{test}_1})_{\text{induced}}\) yields the same current distribution as on the transmission line \((I_{\text{test}})\). Thus, comparing (6) with (8) yields

\[
I_{\text{pp}} = \frac{L_{11} I_{\text{test}} - \sum_{k=2}^{N} (L_{1k} - L_{11})I_{\text{pl}_k}}{\sum_{k=1}^{N} (L_{1k})} \tag{9}
\]

Nevertheless, it must be pointed out that (9) is based on quasi-static fields to be dominant as it inherits TICE approximations. Further, knowledge of the push-pull mode current for the complete characterization is also required.

V. PUSH-PULL MODE CURRENT

TICE equations in [15] are used within the locality of the bundle to solve for push-pull mode currents (see Fig. 6). Namely, field coupling to return and \(i^{th}\) conductors in the bundle is formulated as follow:

\[
\frac{dV_{\text{ref}_i}}{dx} + j\omega L_{1i}I_1 + \ldots + j\omega L_{1N}I_N = E_{1x}^{\text{exc}} + j\omega A_{\text{ns}_1} \tag{10}
\]

\[
\frac{dV_{\text{ref}_i}}{dx} + j\omega L_{1i}I_1 + \ldots + j\omega L_{1N}I_N = E_{1x}^{\text{exc}} + j\omega A_{\text{ns}_1} \tag{11}
\]

where \(E_{\text{exc}}^{\text{exc}}\) and \(E_{1x}^{\text{exc}}\) are the tangential electric fields along the \(i^{th}\) and 1st transmission lines respectively. They are computed in the absence of the bundle and \(A_{\text{ns}}\) refers to non-static vector potential contributions.

Fig. 6. Quasi-static current decomposition for push-pull mode analysis

For (10) and (11), the return conductor is the surrounding structure. The per unit length inductance is computed with the assumption that the surrounding structure is the reference conductor. However, we need a new set of coupling equations so that conductor with subscript “1” in the bundle is the return conductor. To do so, we subtract (11) from (10) to obtain

\[
\frac{d(V_{\text{ref}_i} - V_{\text{ref}_f})}{dx} + j\omega (L_{1i} - L_{11})I_1 + \ldots + j\omega (L_{1N} - L_{1N})I_N = E_{1x}^{\text{exc}} - E_{1x}^{\text{exc}} + j\omega (A_{\text{ns}_1} - A_{\text{ns}_1}) \tag{12}
\]

Next, replacing the currents in (12) with their equivalent push-pull and push-push mode currents and using the method demonstrated in Fig. 6. (12) gives

\[
\frac{d(V_{\text{ref}_i} - V_{\text{ref}_f})}{dx} + j\omega (L_{1i} - L_{11})(I_{\text{pp}} - I_{\text{pl}_N}) + \ldots = E_{1x}^{\text{exc}} - E_{1x}^{\text{exc}} + j\omega (A_{\text{ns}_1} - A_{\text{ns}_1}) \tag{13}
\]

Restructuring the terms in (13) further yields

\[
\frac{d(V_{\text{ref}_i} - V_{\text{ref}_f})}{dx} + j\omega \sum_{k=1}^{N} (L_{1k} - L_{11})I_{\text{pp}} + j\omega L_{1T}[l_{pl}] = E_{1x}^{\text{exc}} - E_{1x}^{\text{exc}} + j\omega (A_{\text{ns}_1} - A_{\text{ns}_1}) \tag{14}
\]
where
\[ \tilde{L}_T(i, j) = L_{i+1,j+1} - L_{i,j+1} - L_{i+1,1} + L_{i,1} \]

\[ [I_{pl}] = [I_{pl1} \cdots I_{pl,i} \cdots I_{pl,N}]^T \]

The second term \( \sum_{k=1}^{N} (L_{ik} - L_{ik}')I_{pp} \) in (14) can be associated with the non-static vector potential difference between highly coupled wires in which quasi-static mode is still the dominant mode. Namely, for wires in close proximity, the following relation holds,

\[ \sum_{k=1}^{N} (L_{ik} - L_{ik}')I_{pp} \approx A_{n_i} - A_{n_{i+1}} \quad (15) \]

Thus, (14) reduces to

\[ \frac{d(V_{re_{ref_i}} - V_{re_{ref_{i+1}}})}{dx} + j \omega \tilde{L}_T[I_{pl}] = E_{t_{ref}}^{exc} - E_{t_{ref}}^{exc} \quad (16) \]

In a similar manner, we obtain Telegrapher’s second coupling equation as

\[ \frac{d[I_{pl}]}{dx} + j \omega \tilde{C}_T[V_{re_f} - V_{re_{ref_1}}] = 0 \quad (17) \]

where

\[ \tilde{C}_T = \mu \tilde{L}_T^{-1}, \quad [V_{re_{ref_i}} - V_{re_{ref_{i+1}}} = \begin{bmatrix} V_{re_{ref_2}} - V_{re_{ref_1}} \\ \vdots \\ V_{re_{ref_i}} - V_{re_{ref_{i-1}}} \\ : \\ V_{re_{ref_N}} - V_{re_{ref_1}} \end{bmatrix} \]

Equations (16) and (17) constitute the final form of the multiconductor transmission line formulation for the whole bundle with conductor “1” being the return/reference conductor.

VI. ANALYSIS TOOLS AND ALGORITHM

The rigorous evaluation of (9) requires full wave solvers and furthermore the associated source terms in (16) and (17) must be precomputed in the absence of the transmission lines via full wave solvers. During the course of this work, we employed our own Multilevel Fast Multipole Method (MLFMM) code EMCAR (see [20], [21] and [22]) to model the test wire with the surrounding structure. EMCAR is also used to validate the proposed method by modeling the exact surrounding geometry. SPICE-like models are then simulated via SLSIM, a network simulator developed by Simlab as well.

The algorithm outlined in table I is followed to implement the proposed method.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SIMULATION ALGORITHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Three transmission lines inside a trough</td>
<td></td>
</tr>
<tr>
<td>1. Model the whole geometry in CableMod and EMCAR</td>
<td></td>
</tr>
<tr>
<td>2. Run EMCAR with test wire to compute push-push mode current from (9)</td>
<td></td>
</tr>
<tr>
<td>3. Run CableMod to compute push-pull mode current from (16) and (17)</td>
<td></td>
</tr>
<tr>
<td>4. Use the composition matrix in Fig. 3 to obtain the total current induced on each transmission line</td>
<td></td>
</tr>
</tbody>
</table>

A. Three transmission lines inside a trough

We first examine the geometry in Fig. 8 showing three transmission lines inside a trough. The entire structure is illuminated by a plane wave incident at \((\theta, \phi) = (45^\circ, 45^\circ)\) as shown. The frequency is 0.6GHz and the structure is 3mx1mx0.6m with the wires being 2.1m long.

![Fig. 8. Three transmission lines inside a trough illuminated by plane wave](image)

The wire current for each conductor is shown in Fig. 9. As seen, the MTLT current is significantly different than the total current induced on each transmission line since it does not account for non-static contributions peculiar to the geometry. However, the proposed technique yields nearly identical results to those of the full wave method where no decomposition between the wires and structure was applied.

B. Three transmission lines inside a cylindrical cavity

To validate the proposed method in cavity-like structures, we next consider the geometry in Fig. 10, displaying three transmission lines inside a cylindrical cavity illuminated by

![Fig. 7. Equivalent representation of a wire under the thin wire assumption](image)
C. Three transmission lines within an automobile

We next proceed to consider a more realistic structures such as that in Fig. 12, displaying three transmission lines located inside an automobile. The automobile is about 5m long, 1.74m wide and 1.16m tall. Again the entire structure enclosing the wires was illuminated with a plane wave operating at 0.4GHz.

As compared to the conventional MTLT in Fig. 13, our solution is very close to the rigorous one since it accounts for non-static contributions via the test wire analysis.

D. Five transmission lines within an automobile structure

As a more complex example, we add more wires to the bundle as in Fig. 14. Specifically, we consider 5 wires with the whole structure illuminated by a plane wave at 0.3GHz.

Failure of the conventional MTLT for highly complex structures is better substantiated in this example as shown in the results of Fig. 15. We observe that the proposed method agrees very well with rigorous data even for highly non-static contributions.
VIII. DISCUSSION

The proposed technique is an alternative approach to TICE to capture the non-static contributions via a more computationally efficient approach. It employs transmission line theory within the bundle whereas the analysis between the structure and the bundle is done using a test wire. As it inherits TICE in the push-push mode analysis and MTLT for push-pull mode characterization, the restrictions peculiar to TICE and MTLT also apply to the proposed method. For instance, the proposed method fails at transmission line resonances.

The proposed method also requires quasi-static currents to be dominant, as a limitation inherent to TICE. Of course, due to the complexities in realistic problems, it is often difficult to find whether quasi-static contribution constitutes the dominant current. To extract that the quasi-static contributions dominant, we propose a comparison of the test wire current with quasi-static current found via MTLT analysis of a single wire within the same geometry.

IX. CONCLUSION

A computationally efficient method for field coupling analysis of multiconductor transmission lines in presence of complex structures was presented. The proposed method employed a test wire analysis to evaluate the coupling between the wire bundle and the surrounding structure. Subsequently, transmission line analysis was employed to find the perturbation current within the bundle.

The test wire analysis involved use of full wave solvers to compute the push-push mode (average) current in the bundle whereas the push-pull mode current analysis employed CableMod software to generate SPICE-like model of the whole bundle. This was done by taking one of the conductors as a reference conductor within the bundle.

Several examples were given to show that the traditional transmission line theory fails with increasing complexity of the surrounding structure. At the same time, it was shown that out method remained valid for all cases regardless of structure complexity and the number of wires within the bundle.

ACKNOWLEDGMENT

The authors would like to thank Dr. Kubilay Sertel of The Ohio State University for his useful comments and suggestions.

REFERENCES


